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**COMPUTING PROGRAMS FOR THE  
COMPLEX FRESNEL INTEGRAL**

by

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ABSTRACT

The asymptotic series for the complex Fresnel integral with remainder is used in subroutines to evaluate the conventional Fresnel integrals and the probability integral. A series expansion with improved accuracy of calculation is used to evaluate the remainder.

A rational polynomial approximation is developed for the complex Fresnel integral. The range of validity of the rational approximation is that part of the complex plane on the negative side which is bounded by the imaginary axis and is outside a circle of unit radius.

The complex Fresnel integral correct to thirteen significant digits is tabulated at unit intervals in the argument.

FOREWORD

The work which is covered by this report was performed in the Mathematical Physics Branch partly under the Foundational Research Program of the Naval Weapons Laboratory and partly under a project established at the Naval Weapons Laboratory by the Office of Naval Research (ONR Project NR 062-203). Assistance in the programming was contributed by E. J. Hershey and by W. H. Langdon. The date of completion was 25 January 1962.

APPROVED FOR RELEASE:

/s/ R. H. LYDDANE  
Technical Director

# INTRODUCTION

The Fresnel integrals and the probability or error integral have a common origin insofar as they can be generated from a single complex integral. Various forms of complex integral with different constants of integration may be utilized. Throughout the present report the complex Fresnel integral  $E(z)$  for complex argument  $z$  will be defined by the equation

$$E(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z \frac{e^t}{t^{\frac{1}{2}}} dt \quad (1)$$

where the path of integration lies within that part of the complex plane from which the positive real axis is excluded. The phase of  $z$  is limited to the range 0 to  $2\pi$ , and the phase of  $z^{\frac{1}{2}}$  is half the phase of  $z$ .

The conventional Fresnel integrals are defined in terms of harmonic functions by the equations

$$C(v) + iS(v) = \int_0^v e^{\frac{1}{2}\pi i u^2} du = \int_0^v i u^2 du \quad (2)$$

or are expressed in terms of Hankel functions by the equations

$$C(x) + iS(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \frac{e^{iu}}{u^{\frac{1}{2}}} du = \frac{i}{2} \int_0^x H_{\frac{1}{2}}^{(1)}(u) du \quad (3)$$

where  $x$  and  $v$  are related by the equation

$$x = \frac{1}{2} \pi v^2 \quad (4)$$

The conventional integrals are recovered from the complex integral through the transformations

$$t = iu \quad z = ix \quad (5)$$

and are expressed by the equation

$$C(x) + iS(x) = \frac{1}{2} + \frac{i}{2} + \frac{(1-i)}{\sqrt{2}} E(ix) \quad (6)$$

The transformations

$$t = -u^2 \quad z = -x^2 \quad (7)$$

give the error function of real argument,

$$H(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du = 1 - i\sqrt{2} E(-x^2) \quad (8)$$

and the transformations

$$t = +u^2 \quad z = +x^2 \quad (9)$$

give the error function of imaginary argument,

$$-iH(ix) = \frac{2}{\sqrt{\pi}} \int_0^x e^{u^2} du = i + \sqrt{2} E(+x^2) \quad (10)$$

Values of the Fresnel integrals and the probability integral are to be found in a few published tables.<sup>1-4\*</sup> It is believed that Fresnel integrals of complex argument have not appeared heretofore.

Tables are too bulky for use on a high speed calculator, and a compact subroutine is generally to be preferred for machine computations.

The complex Fresnel integral is given by an absolutely convergent series for all values of the argument, but for large arguments the terms of the series increase with order before they finally decrease.

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\* Superscripts in the text refer to similarly numbered entries in the list of references.

The rounding error in the sum is contributed by the largest terms and increases with increase of argument. The integral has an asymptotic series where the terms decrease with order at first but finally increase. The truncation error diminishes with increase in argument. There is an intermediate range of argument where neither series is effective. This region of difficulty can be spanned by a rational polynomial approximation. Insofar as the complex Fresnel integral and its rational approximation are both analytic throughout a finite region in the complex plane, the error in the approximation is also analytic in the same region. The absolute value of an analytic function within a closed boundary is everywhere less than the maximum absolute value of the function on the boundary. The approximation therefore is so adjusted as to make the error less than a tolerance everywhere on the boundary of the region of approximation.

A rational approximation which is valid over the negative half of the complex plane outside a circle of unit radius has been so constructed by the method of inverted differences as to have an absence of error at nodal points on the boundary. Between each pair of nodal points there was a maximum of error, and the nodal points were so adjusted as to equalize the maxima of error. The distribution of error was thus optimized in accordance with the Chebyshev criterion.

In previous work <sup>7</sup>, the adjustment of spacing between nodes was computed solely from the intermediate maximum of error. There was a seichelike instability in the motion of the nodes. In current work, the adjustment of each node is derived from an entire matrix. The instability is no longer apparent in the motion of the nodes.\*

A smaller bound of error could have been achieved if the error had been allowed to be finite instead of null between the maxima. The rational approximation then would be specified at antinodal points in

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\* These methods will be documented in a later report.

the error curve. An antinodal specification has been used successfully on the real axis by Murnaghan and Wrench.<sup>5</sup> The problem of controlling a profusion of maxima along a complex contour looks formidable.

The rational approximation has been incorporated in a subroutine for use on the Naval Ordnance Research Calculator. Two additional subroutines give the conventional probability integral or give the conventional Fresnel integrals. Formulations and instructions for the subroutines are presented in the present report.

The NORC does floating decimal arithmetic with 13 digit numbers at the average rate of 15000 operations per second in response to three address instructions.

General purpose subroutines in the current programming are designed to confine the error in a function to the thirteenth digit unless one unit error in the last digit of the argument would be responsible for a larger inherent error in the function. Inasmuch as the majority of terms in a series expansion have decimal exponents at least one unit lower than the decimal exponent of the principal terms, the smaller terms are summed first and the principal terms are added last.

The sequence of computations is arranged as far as possible so that the decimal coefficients of all numbers are just less than unity where the relative rounding error is least.

#### ANALYSIS

The path of integration in Equation (1) may be varied in accordance with the equation

$$E(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 \frac{e^t}{t^{\frac{1}{2}}} dt + \frac{1}{\sqrt{2\pi}} \int_0^z \frac{e^t}{t^{\frac{1}{2}}} dt \quad (11)$$



The integrand in Equation (11) can be expanded as an ascending power series. Term by term integration of the series leads to the equation

$$E(z) = -\frac{i}{\sqrt{2}} + \left(\frac{2z}{\pi}\right)^{\frac{1}{2}} \sum_{m=0}^{\infty} \frac{z^m}{(2m+1)m!} \quad (12)$$

The series in Equation (12) is absolutely convergent for all values of  $z$ , but the largest term in the series increases with increase in  $|z|$ .

Successive integrations by parts in Equation (1) lead to the equation

$$E(z) = \frac{e^z}{(2\pi z)^{\frac{1}{2}}} \sum_{m=0}^{m=N-1} \frac{(2m)!}{2^{2m} m!} z^m + R(z) \quad (13)$$

in which the remainder  $R(z)$  is defined by the equation

$$R(z) = \frac{(2N)!}{\sqrt{2\pi} 2^{2N} N!} \int_{-\infty}^z \frac{e^t}{t^{N+\frac{1}{2}}} dt \quad (14)$$

The series in Equation (13) is asymptotic and the error in the series is not much smaller than the smallest term. Improved accuracy is achieved if the series is terminated with half the smallest term. Evaluation of the remainder would remove the error in the asymptotic series. A term by term comparison between Equations (12) and (13) establishes a series expansion for the remainder, which is given by the equation

$$R(z) = -\frac{i}{\sqrt{2}} + \left(\frac{2z}{\pi}\right)^{\frac{1}{2}} \frac{(2N)!}{2^{2N} N!} \sum_{m=0}^{m=\infty} \frac{z^{m-N}}{(2m+1-2N)m!} \quad (15)$$

The last term of the series in Equation (15) is a minimum if  $N$  is so chosen that

$$N - \frac{3}{2} < |z| < N - \frac{1}{2} \quad (16)$$

The terms of the series in Equation (15) then increase until  $m = N - 1$  then decrease after  $m = N$ . The largest terms in the series are on the order of unity, and the accuracy with which  $R(z)$  can be computed is essentially constant.

The inherent errors in Equations (12) and (13) are compared in Figure 1, Appendix A. Curve E represents the Fresnel integral itself on the negative real axis. Curve H illustrates the rounding error in the largest term of the absolutely convergent series. Curve A illustrates the smallest term in the asymptotic series. Curve R illustrates the accuracy which could be achieved by a rational approximation.

Accurate values of the complex Fresnel integral for equally spaced arguments have been calculated with double precision by direct integration in the complex plane. Sixteen point Gaussian integration was used for each unit interval. The integration multipliers for the Gaussian integration were taken from Reference 6. The overall accuracy of the integrations was checked against double precision values which were computed from the ascending power series at  $|z| = 1$ . The double precision values have been rounded to thirteen digits and the rounded values are tabulated in Table 5, Appendix B.

The zone of approximation for the complex Fresnel integral is the area A in Figure 4. The infinite zone of approximation was reduced to a finite zone of approximation by the inversion

$$\frac{1}{z} = \frac{1}{z} \quad (17)$$

which exchanges the area A for the area B in Figure 4. The complex Fresnel integral  $E(z)$  was exchanged for the monotonic function  $F(\bar{z})$  which is defined by the equation

$$F(\bar{z}) = (2\pi z)^{\frac{1}{2}} e^{-z} E(z) \quad (18)$$

The function  $F(\bar{z})$  is expressed in terms of two polynomials  $p(\bar{z})$  and  $q(\bar{z})$  by the equation

$$F(\bar{z}) = \frac{p(\bar{z})}{q(\bar{z})} \quad (19)$$

while the polynomials themselves are defined by the equations

$$p(\bar{z}) = \sum_{m=0}^{m=N} a_m \bar{z}^m \quad (20)$$

$$q(\bar{z}) = \sum_{m=0}^{m=N} c_m \bar{z}^m \quad (21)$$

in the rational polynomial approximation of the  $N$ th degree there are  $2N + 1$  complex coefficients which are specified by the  $2N + 1$  complex values of the function to be approximated, at  $2N + 1$  nodes on the half circle contour of B. The function  $F(\bar{z})$  is symmetric with respect to the real axis and the polynomials  $p(\bar{z})$  and  $q(\bar{z})$  have real coefficients. Values of the function on the positive quarter circle are sufficient to establish the real coefficients. The coefficients of  $p(\bar{z})$  and  $q(\bar{z})$  were determined by a double precision complex subroutine which used the method of inverted differences.

The origin at  $\bar{z} = 0$  and the corners at  $\bar{z} = \pm i \frac{\pi}{2}$  were included among the nodes, and since there must be an odd number of nodes, the point at  $\bar{z} = -1$  could not be a node. For each order of approximation the locations of the other nodes were adjusted until the maxima of

error between nodes were uniform. The number of nodes on the imaginary axis or the number of nodes on the half circle was increased according to whichever line had the largest maxima of error until  $N = 13$ .

The function  $F(\bar{z})$  was finally approximated to within relative errors as listed in the following table:

<u>Line</u>	<u>Number of Nodes</u>	<u>Maximum Error Between Nodes</u>
Imaginary Axis	19	$5.6 \times 10^{-13}$
Half Circle	8	$2.5 \times 10^{-13}$

The variation of error along the contour of approximation is plotted in Figure 5.

The coefficients of the polynomials and useful ratios between the coefficients are listed to thirteen digits in Tables 1 to 4, Appendix B.

That the error in the rational polynomial approximation is truly analytic within the zone of approximation is verified by a determination of the roots of  $q(\frac{1}{z})$ , which are all found to lie outside the zone of approximation. The roots are represented by dots in Figure 4.

The rational polynomial approximation and the absolutely convergent series have been extrapolated beyond their zones of validity to an empirical boundary where the two approximations give comparable accuracy. The accuracy of this extrapolation is at worst one digit less than the inherent accuracy for one unit error in the thirteenth digit of the argument. The relative errors in the range of worst error are illustrated in Figure 6. The errors in Figure 6 are actual values for a subroutine which uses the rational polynomial in areas A + D but uses the convergent series in areas B + C of Figure 4.

# PROBABILITY INTEGRAL ROUTINE

## Operation

The routine computes the probability integral from a given argument.

## Formulation

The probability integral or error function  $H(x)$  is calculated for either positive or negative argument  $x$  with the equation

$$H(x) = -\frac{e^{-x^2}}{x\sqrt{\pi}} \sum_{m=0}^{N-1} \frac{(-1)^m (2m)!}{2^{2m} m! x^{2m}} + R(x) \quad (22)$$

The summation in Equation (22) is not cycled if  $x^2 < \frac{3}{2}$ . Otherwise the summation is cycled until  $x^2 < N + \frac{3}{2}$ , or until the terms in the summation become less than  $10^{-18}$ . The order  $N$  is even because the terms are computed in pairs.

If the final asymptotic term is more than  $10^{-18}$ , then the remainder  $R(x)$  is computed with the equation

$$R(x) = \frac{2}{\sqrt{\pi}} \frac{(2N)!}{2^{2N} N!} \sum_{k=0}^{N-1} \frac{(-1)^k x^{2k+1-2N}}{(2k+1-2N)k!} \\ + \frac{2}{\sqrt{\pi}} \frac{(2N)!}{2^{2N} N!} \sum_{k=0}^{N-\infty} \frac{(-1)^{N+k} x^{2k+1}}{(2k+1)(N+k)!} \quad (23)$$

Only the last summation in Equation (23) is cycled with  $N = 0$  if  $x^2 < \frac{3}{2}$ . In any case, the last summation is cycled in two parts, for  $k \leq 2$ , and for  $k > 2$ , in order to minimize the accumulation of rounding error. In the combination of summations and remainders the program

utilizes the sequence

$$\left( \frac{2}{\sqrt{\pi}} - 1 \right) \sqrt{\frac{\pi}{2}} R + \Sigma + \sqrt{\frac{\pi}{2}} R \quad (24)$$

whose terms are all less than unity and therefore are free from extreme rounding error.

If the final asymptotic term is less than  $10^{-13}$ , then the remainder  $R(x)$  is given by the equation

$$R(x) = \pm 1 \quad (25)$$

where the sign is the same as the sign of  $x$ .

#### Library Code

Block 0078 in Deck 2500 contains the NORC code. Beginning of block words and assembly instructions are as follows:

Program	BOB	0991	0101	0191	0078
Guide Words	Block	0079			
Subroutines (exp)	BOB	0994	0192	0241	0058

#### Input

- The argument  $x$  in memory location  $X$ .

#### Output

- The function  $H(x)$  in memory location  $Y$ .

#### Call Lines

$L$	0060	$L + 1$	-	0101
$L + 1$	$Y$	$C$	-	$X$

#### Exit Line

The program returns control to  $C$ .

### Limitations

- a. Data on the accuracy are plotted in Figure 2, Appendix A.

### Time

- a. A representative time to compute each value of the function is 32 milliseconds.

## FRESNEL INTEGRAL ROUTINE

### Operation

The routine computes the Fresnel integrals from a given argument.

### Formulation

The Fresnel integrals are computed with the equations

$$S(x) = p(x) \sin x - q(x) \cos x + P(x) \quad (26)$$

$$C(x) = q(x) \sin x + p(x) \cos x + Q(x) \quad (27)$$

in which  $p(x)$  and  $q(x)$  are defined by the equation

$$q(x) + ip(x) = \frac{1}{(2\pi x)^{\frac{1}{2}}} \sum_{m=0}^{N-1} \frac{(2m)!}{2^{2m} m! (ix)^m} \quad (28)$$

The summation in Equation (28) is not cycled if  $x < \frac{3}{2}$ . Otherwise the summation is cycled until  $x < N + \frac{3}{2}$  or until the smallest term in  $p(x)$  is less than  $10^{-13}$ . The order  $N$  is even because the terms are computed in pairs.

If the final term of  $p(x)$  is more than  $10^{-13}$ , then the remainders  $P(x)$  and  $Q(x)$  are evaluated with the equation

$$\begin{aligned} Q(x) + iP(x) = & \left(\frac{2x}{\pi}\right)^{\frac{1}{2}} \frac{(2N)!}{2^{2N} N!} \sum_{k=0}^{N-1} \frac{(ix)^{k-N}}{(2k+1-2N)k!} \\ & + \left(\frac{2x}{\pi}\right)^{\frac{1}{2}} \frac{(2N)!}{2^{2N} N!} \sum_{k=0}^{\infty} \frac{(ix)^k}{(2k+1)(N+k)!} \end{aligned} \quad (29)$$

Only the last summation in Equation (29) is cycled with  $N = 0$  if  $x < \frac{3}{2}$ . In any case, the last summation is cycled in two parts, for  $k \leq 2$  and for  $k > 2$ , in order to minimize the accumulation of rounding error.

If the final term of  $p(x)$  is less than  $10^{-13}$ , then the remainders  $P(x)$  and  $Q(x)$  are given by the equation

$$Q(x) + iP(x) = \frac{1}{2} + \frac{1}{2}i \quad (30)$$

In each cycle of summation a real term and an imaginary term are calculated and are added to the real and imaginary parts of the sum.

#### Library Code

Block 0080 in Deck 2500 contains the NORC code. Beginning of block words and assembly instructions are as follows:

Program	BOB	0991	0101	0225	0080
Guide Words	Block	0081			
Subroutines (sin-cos)	BOB	0994	0226	0274	0010
(sq rt)	BOB	0994	0275	0300	0052

#### Input

- The argument  $x$  in memory location  $X$ .

#### Output

- The function  $S(x)$  in memory location  $S$ .
- The function  $C(x)$  in memory location  $C$ .

#### Call Lines

$L$	0060	$L$	-	0101
$L + 1$	$S$	$C$	-	$X$

#### Exit Line

The program returns control to  $L + 2$ .



### Limitations

a. The argument  $x$  must be positive. Otherwise a program stop will occur in the square root subroutine.

b. Data on the accuracy are plotted in Figure 3, Appendix A.

### Modification

a. If  $(2x/\pi)^{\frac{1}{2}}$  is already in memory location  $V$ , modify location 0112 as follows and jettison block 0052.

0112	0060	$V$	0200	0115
------	------	-----	------	------

### Time

a. A representative time to compute each pair of values of the functions is 68 milliseconds.

## COMPLEX FRESNEL INTEGRAL ROUTINE

### Operation

The routine computes a complex Fresnel integral from a given complex argument.

### Formulation

The complex argument  $z$  is given in terms of its parts  $x$ ,  $y$  by the equation

$$z = x + iy \quad (31)$$

where the phase of  $z$  is taken to be less than  $2\pi$ .

If the arguments  $x$ ,  $y$  satisfy either of the inequalities

$$x^2 + y^2 < 1 \quad -x + .02x^2 + .044y^2 < 0 \quad (32)$$

then the complex Fresnel integral  $E(z)$  is given by the equation

$$E(z) = -\frac{i}{\sqrt{2}} + \left(\frac{2z}{\pi}\right)^{\frac{1}{2}} \sum_{m=0}^{\infty} \frac{z^m}{(2m+1)m!} \quad (33)$$

in which the summation is cycled until there is no change in the sum.

If the arguments  $x, y$  satisfy both of the inequalities

$$x^2 + y^2 \geq 1 \quad -x + .02x^2 + .044y^2 \geq 0 \quad (34)$$

then the complex Fresnel integral  $E(z)$  is given by the equation

$$E(z) = \frac{e^z}{(2\pi z)^{\frac{1}{2}}} \frac{p(\frac{1}{z})}{q(\frac{1}{z})} \quad (35)$$

for which the polynomials  $p(\frac{1}{z})$  and  $q(\frac{1}{z})$  are defined by the equations

$$p(\frac{1}{z}) = \sum_{m=0}^{m=13} a_m (\frac{1}{z})^m \quad (36)$$

$$q(\frac{1}{z}) = \sum_{m=0}^{m=13} c_m (\frac{1}{z})^m \quad (37)$$

The summations in Equations (36), (37) are cycled until  $q(\frac{1}{z})$  remains unchanged or until  $m = 13$ . The summation is accomplished by a computing loop which utilizes the quotients between coefficients in Tables 3 and 4, Appendix B.

### Modification

The routine computes  $E(z) + \frac{i}{\sqrt{2}}$  if instructions are modified as follows

loc	PQ	R	S	T
0208	5025	0251	0252	0290
0243	9660	0252	0290	0209

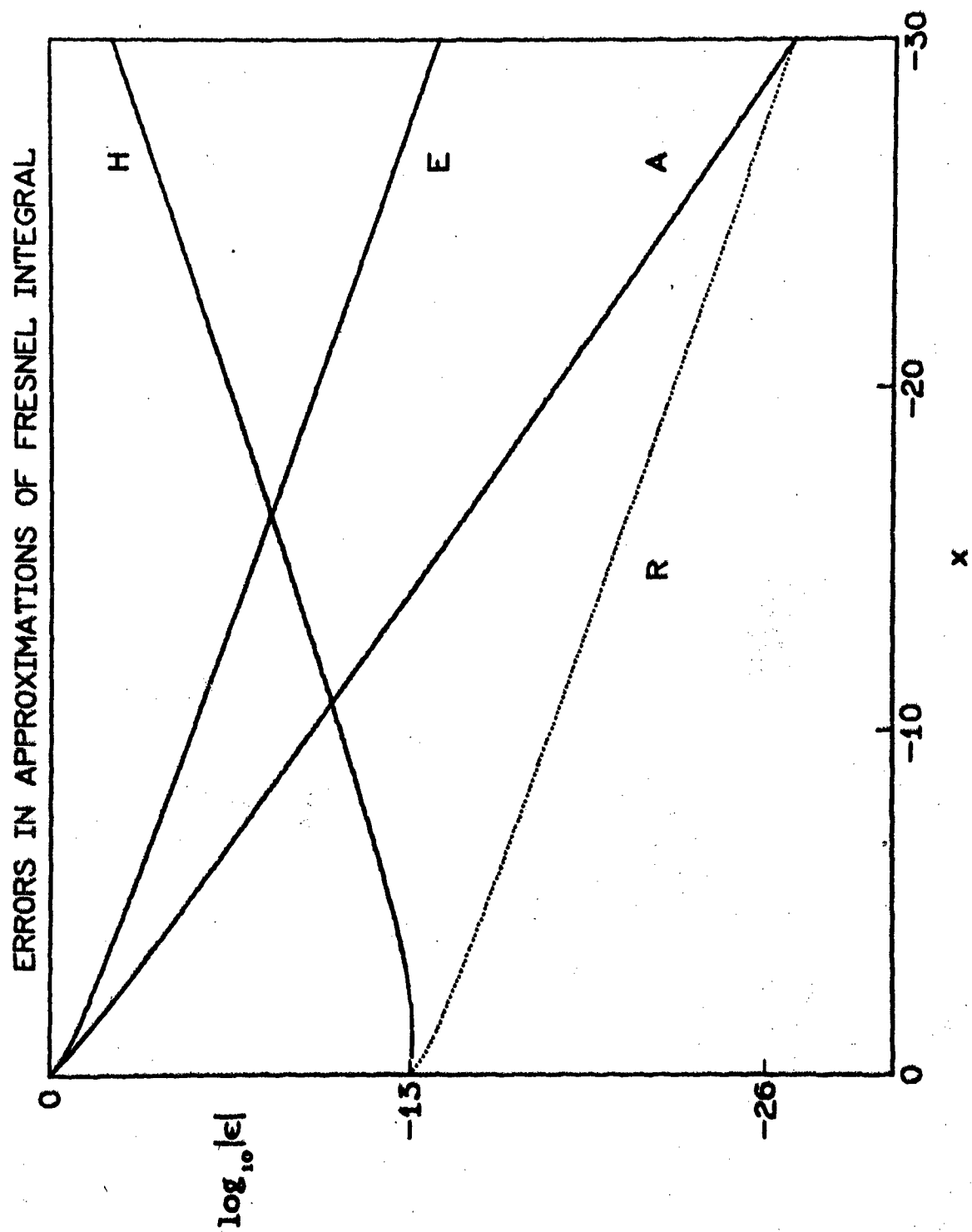
### CONCLUSION

The complex Fresnel integral can be evaluated over the entire range of argument by polynomial approximations to within the inherent error of the argument without resorting to double precision.

### REFERENCES

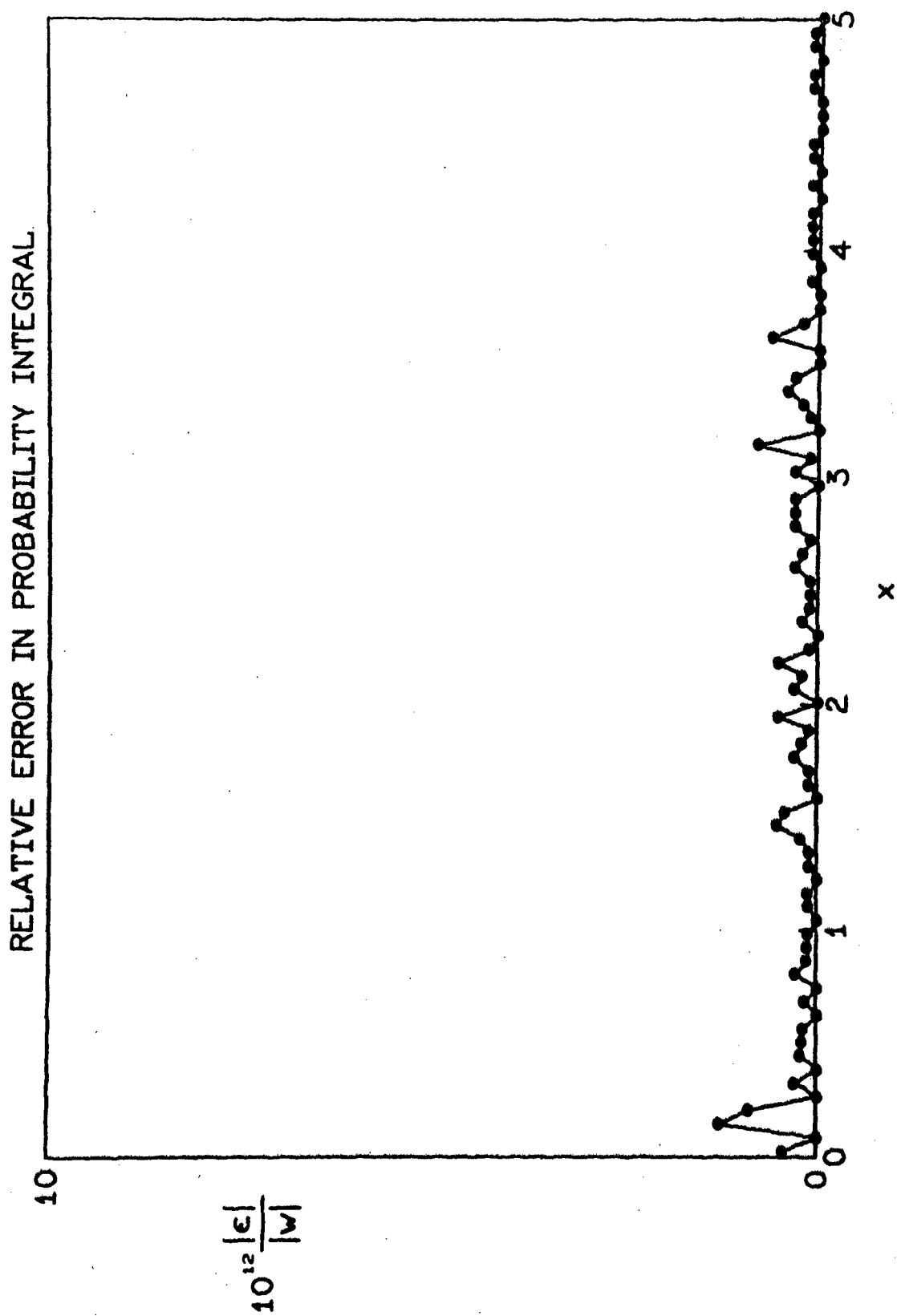
1. *Theory of Bessel Functions*, G. N. Watson (Cambridge University Press, 1944).
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## APPENDIX A



$\epsilon$  is error in  $E(x)$

Figure 1

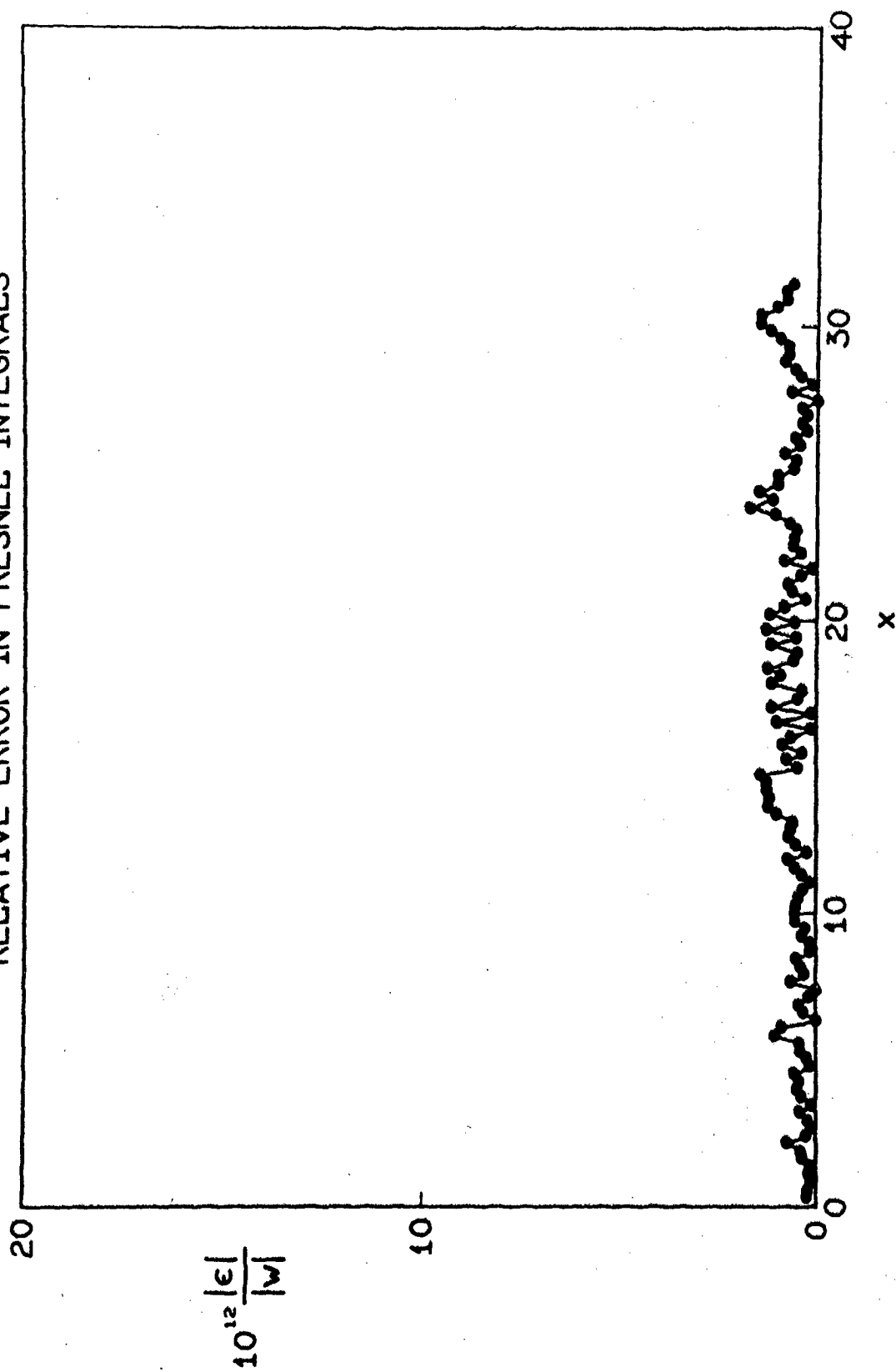


$$w = H(x)$$

$\epsilon$  is error in  $w$  as calculated by subroutine 0078

Figure 2

# RELATIVE ERROR IN FRESNEL INTEGRALS



$$w = C(x) + iS(x)$$

$\epsilon$  is error in  $w$  as calculated by subroutine 0080

Figure 3

# ZONES OF APPROXIMATION OF COMPLEX FRESNEL INTEGRAL

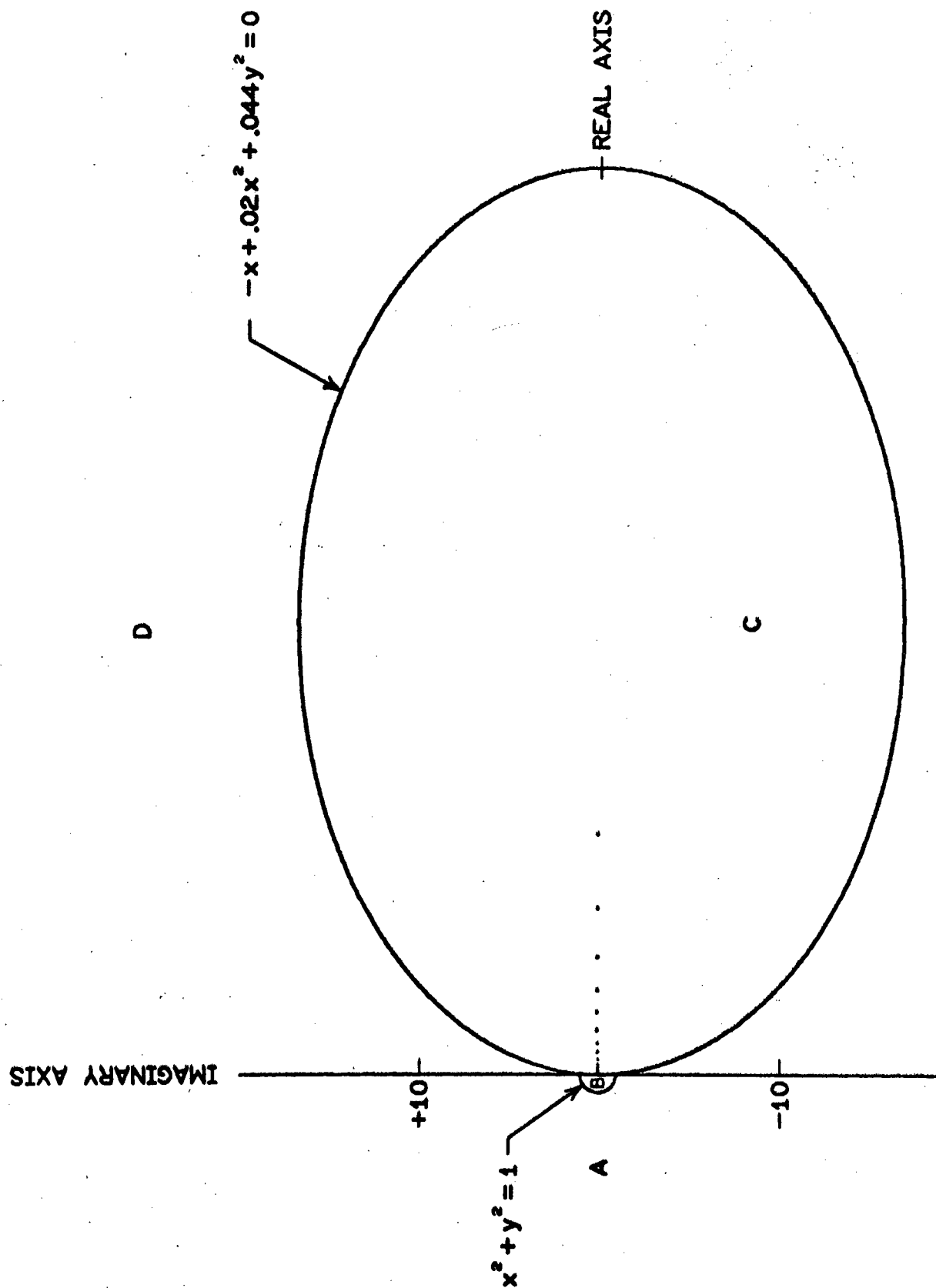
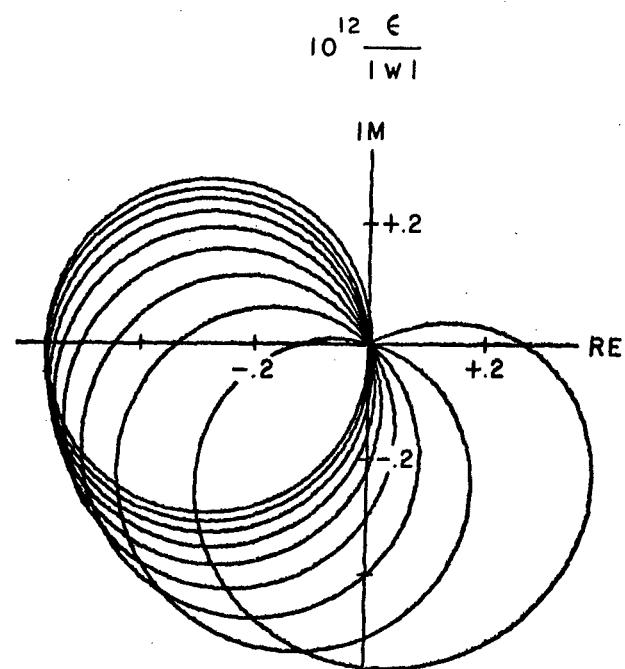


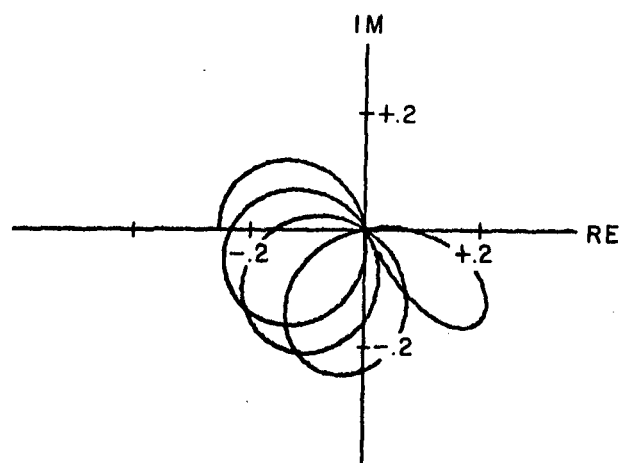
Figure 4.



# RELATIVE ERROR IN RATIONAL POLYNOMIAL APPROXIMATION



a. On imaginary axis.



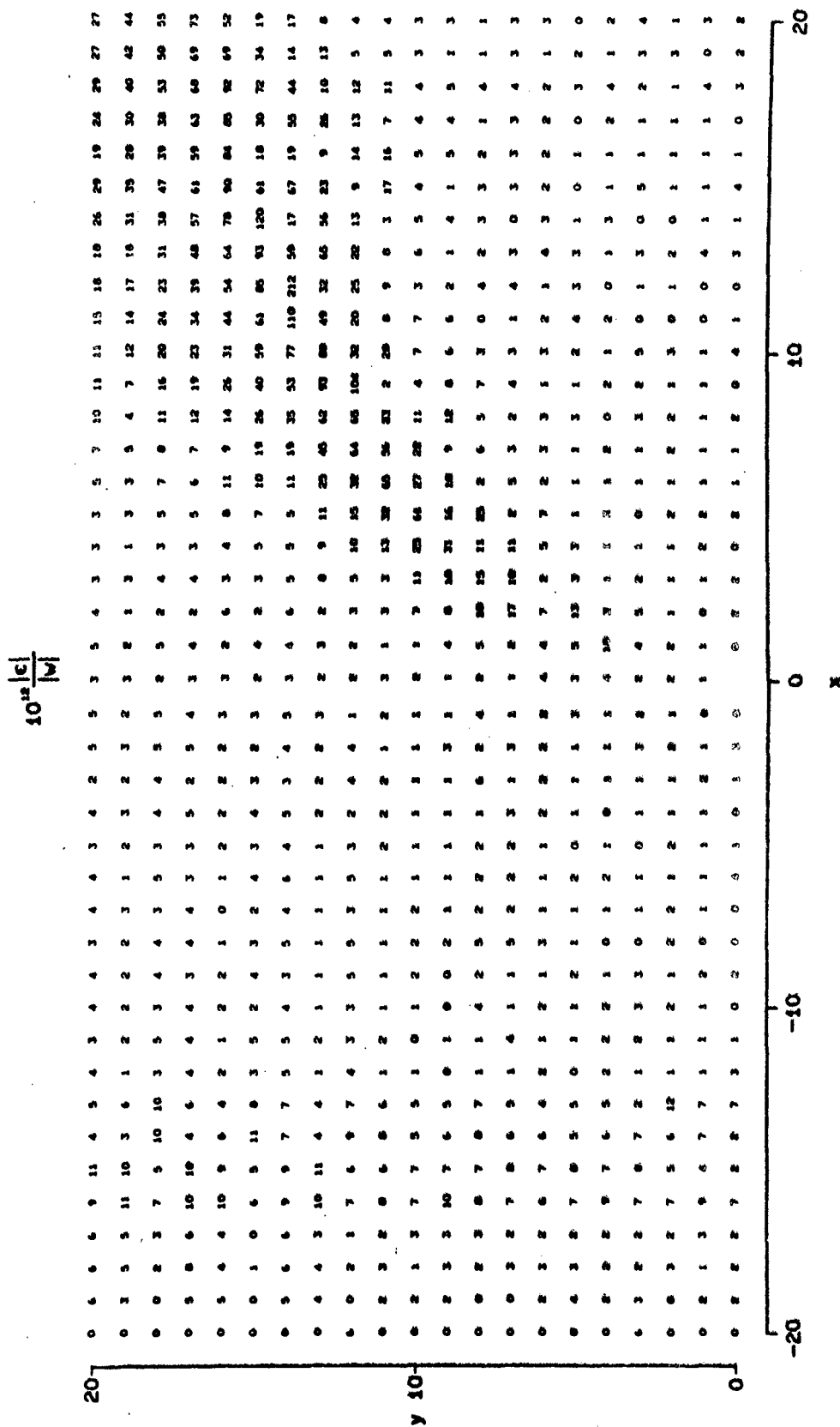
b. On circle  $|z| = 1$ .

$$w = E(z)$$

$\epsilon$  is the error in the double precision approximation of  $w$ .

Figure 5.

# RELATIVE ERROR IN COMPLEX FRESNEL INTEGRAL



$$w = E(z)$$

$\epsilon$  is error in  $w$  as calculated by subroutine 0148

$$z = x + iy$$

Figure 6.

**APPENDIX B**

### TABLES

Each number in the tables contains the sign of the number and the decimal coefficient, followed by the exponent of ten by which the decimal coefficient is to be multiplied.

TABLE 1

COEFFICIENTS OF THE POLYNOMIAL  $p(1/z)$

$a_0$	+ 1.0000 0000 0000	0
$a_1$	- 4.3072 3024 0996	1
$a_2$	+ 7.5767 0484 7754	2
$a_3$	- 7.1538 6843 1803	3
$a_4$	+ 4.0107 3550 9019	4
$a_5$	- 1.3957 8448 9703	5
$a_6$	+ 3.0642 1593 2238	5
$a_7$	- 4.2299 8816 6090	5
$a_8$	+ 3.5959 8801 4722	5
$a_9$	- 1.8050 4449 9502	5
$a_{10}$	+ 4.9680 4778 3506	4
$a_{11}$	- 6.5607 3082 6910	3
$a_{12}$	+ 3.1540 1992 8001	2
$a_{13}$	- 2.3387 8052 9458	0

TABLE 2

COEFFICIENTS OF THE POLYNOMIAL  $q(1/z)$

$c_0$	+	1.0000 0000 0000	0
$c_1$	-	4.3572 3024 0991	1
$c_2$	+	7.7870 6635 9836	2
$c_3$	-	7.5124 1752 2806	3
$c_4$	+	4.3354 6694 4864	4
$c_5$	-	1.5682 5133 2873	5
$c_6$	+	3.6241 8007 0887	5
$c_7$	-	5.3555 3415 7881	5
$c_8$	+	4.9855 2523 2833	5
$c_9$	-	2.8296 1157 2383	5
$c_{10}$	+	9.2431 4330 3151	4
$c_{11}$	-	1.5724 9356 9766	4
$c_{12}$	+	1.1539 4785 3548	3
$c_{13}$	-	2.3345 8467 6245	1

TABLE 3

RATIOS BETWEEN THE COEFFICIENTS OF POLYNOMIAL  $q(1/z)$

$c_0$	+ 1.0000 0000 0000	0
$c_1/c_0$	- 4.3572 3024 0991	1
$c_2/c_1$	- 1.7871 5971 5954	1
$c_3/c_2$	- 9.6473 0127 5810	0
$c_4/c_3$	- 5.7710 6761 1327	0
$c_5/c_4$	- 3.6172 6050 0005	0
$c_6/c_5$	- 2.3109 6890 8438	0
$c_7/c_6$	- 1.4777 2297 5440	0
$c_8/c_7$	- 9.3091 0920 5281	- 1
$c_9/c_8$	- 5.6756 5389 8507	- 1
$c_{10}/c_9$	- 3.2665 7672 5005	- 1
$c_{11}/c_{10}$	- 1.7012 5412 7727	- 1
$c_{12}/c_{11}$	- 7.3383 3114 3189	- 2
$c_{13}/c_{12}$	- 2.0231 2840 1399	- 2

TABLE 4

RATIOS BETWEEN THE COEFFICIENTS OF POLYNOMIALS  $p(1/z)$  AND  $q(1/z)$

$a_0/c_0$	1.0000 0000 0000	0
$a_1/c_1$	9.8852 4820 3035	- 1
$a_2/c_2$	9.7298 5781 5047	- 1
$a_3/c_3$	9.5227 2475 5494	- 1
$a_4/c_4$	9.2509 8855 5619	- 1
$a_5/c_5$	8.9002 6018 4351	- 1
$a_6/c_6$	8.4549 2186 4542	- 1
$a_7/c_7$	7.8983 4970 9645	- 1
$a_8/c_8$	7.2128 5691 4333	- 1
$a_9/c_9$	6.3791 2467 2512	- 1
$a_{10}/c_{10}$	5.3748 4665 1801	- 1
$a_{11}/c_{11}$	4.1721 8292 8473	- 1
$a_{12}/c_{12}$	2.7332 4303 0268	- 1
$a_{13}/c_{13}$	1.0017 9725 8954	- 1



TABLE 5

$$E(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z \frac{e^{-t^2}}{t^2} dt$$

$$z = x + iy$$

$$x = -20 \quad (1) \quad +20$$

$$y = 0 \quad (1) \quad 20$$

COMPLEX FRESNEL INTEGRAL TABLE

		X															
Y		-20				-19				-18				-17			
0	RE	0.0000	0000	0000	0	0.0000	0000	0000	0	0.0000	0000	0000	0	0.0000	0000	0000	0
	IM	-1.7957	8859	7310	-10	-5.0024	0083	0536	-10	-1.3952	4562	8082	-9	-3.8970	1202	0407	-9
1	RE	1.5329	5041	0689	-10	4.2730	9271	7164	-10	1.1927	0706	7775	-9	3.3340	0967	3875	-9
	IM	-9.3340	0781	8312	-11	-2.5948	3436	4177	-10	-7.2210	8370	6848	-10	-2.0118	2027	2982	-9
2	RE	1.5918	5209	8689	-10	4.4278	0670	5481	-10	1.2329	5429	2165	-9	3.4373	4598	0840	-9
	IM	8.2231	8981	2346	-11	2.3006	8423	0491	-10	6.4476	6350	8442	-10	1.8103	5180	3243	-9
3	RE	1.2573	5485	7283	-11	3.3256	2654	8503	-11	8.7312	9320	2370	-11	2.2701	2189	3428	-10
	IM	1.7822	3261	4560	-10	4.9632	5675	6223	-10	1.3838	4074	2679	-9	3.8634	3726	4057	-9
4	RE	-1.4505	3894	1195	-10	-4.0502	1859	1169	-10	-1.1325	1006	0329	-9	-3.1716	4037	7657	-9
	IM	1.0311	8917	3671	-10	2.8511	8578	9746	-10	7.8863	6211	7789	-10	2.1821	3007	2314	-9
5	RE	-1.6278	5844	7564	-10	-4.5172	2955	9413	-10	-1.2544	5976	7338	-9	-3.4864	7949	7232	-9
	IM	-6.9750	0634	4092	-11	-1.9658	8927	5626	-10	-5.5525	6253	7918	-10	-1.5719	7277	0464	-9
6	RE	-2.5205	5776	6984	-11	-6.6862	5922	5954	-11	-1.7625	1206	0850	-10	-4.6084	0721	4154	-10
	IM	-1.7424	8178	5860	-10	-4.8489	5279	9649	-10	-1.3507	1155	0683	-9	-3.7665	4520	8106	-9
7	RE	1.3457	1025	6296	-10	3.7621	7313	9620	-10	1.0531	9948	9469	-9	2.9526	6424	4656	-9
	IM	-1.1167	5491	2819	-10	-3.0746	2576	8472	-10	-8.4637	6923	7769	-10	-2.3292	8309	5963	-9
8	RE	1.6427	3253	6806	-10	4.5479	8836	5283	-10	1.2596	7801	5317	-9	3.4904	2928	8103	-9
	IM	5.5985	3204	1569	-11	1.5923	3990	7171	-10	4.5391	0808	3527	-10	1.2970	3119	0591	-9
9	RE	3.7871	0993	7253	-11	1.0085	9083	9096	-10	2.6727	9988	4962	-10	7.0391	8173	4777	-10
	IM	1.6789	3339	2442	-10	4.6675	1263	2360	-10	1.2985	8386	1796	-9	3.6157	3307	0223	-9
10	RE	-1.2211	3745	2449	-10	-3.4172	6307	9356	-10	-9.5742	1983	5510	-10	-2.6857	3924	3395	-9
	IM	1.1909	1228	3787	-10	3.2688	0375	8467	-10	8.9680	4682	3269	-10	2.4589	0405	2841	-9
11	RE	-1.6385	1964	3164	-10	-4.5272	5015	0577	-10	-1.2511	0408	0707	-9	-3.4578	4007	2842	-9
	IM	-4.1174	0389	2400	-11	-1.1869	5847	3556	-10	-3.4281	7127	8449	-10	-9.9204	3783	9713	-10
12	RE	-5.0429	4443	0611	-11	-1.3486	8584	7745	-10	-3.5935	5571	2645	-10	-9.5320	3256	6766	-10
	IM	-1.5946	8238	1331	-10	-4.4289	3290	8836	-10	-1.2307	2135	1322	-9	-3.4217	6100	8385	-9
13	RE	1.0804	3486	7814	-10	3.0266	9379	3568	-10	8.4870	8726	9993	-10	2.3821	7382	6367	-9
	IM	-1.2537	1596	1978	-10	-3.4344	0755	1428	-10	-9.4025	5864	4532	-10	-2.5722	8526	7856	-9
14	RE	1.6169	1601	9824	-10	4.4605	3268	4760	-10	1.2305	1714	2193	-9	3.3943	8731	7878	-9
	IM	2.5653	1789	8246	-11	7.5994	1527	6357	-11	2.2512	3023	7699	-10	6.6693	2520	3169	-10
15	RE	6.2647	7553	3494	-11	1.6819	7216	4633	-10	4.5035	4846	5212	-10	1.2020	1096	6391	-9
	IM	1.4928	2751	9851	-10	4.1427	5498	5376	-10	1.1500	7602	7336	-9	3.1937	8903	6134	-9
16	RE	-9.2747	2410	2278	-11	-2.6021	4424	2149	-10	-7.3061	6176	7884	-10	-2.0528	8424	6599	-9
	IM	1.3044	7736	1491	-10	3.5694	0441	6092	-10	9.7608	9234	6663	-10	2.6672	6054	7211	-9
17	RE	-1.5790	8775	7500	-10	-4.3512	5048	9116	-10	-1.1988	9356	1337	-9	-3.3027	6424	7480	-9
	IM	-9.7992	8453	3119	-12	-3.2256	7556	0164	-11	-1.0424	6920	5623	-10	-3.3217	6116	9712	-10
18	RE	-7.4248	9134	0410	-11	-2.0001	1467	3073	-10	-5.3773	0911	3851	-10	-1.4424	3823	1906	-9
	IM	-1.3761	3564	4755	-10	-3.8170	5650	7786	-10	-1.0590	0019	4855	-9	-2.9386	2742	1401	-9
19	RE	7.6594	3783	2271	-11	2.1543	9637	0263	-10	6.0630	0236	1063	-10	1.7071	1732	2987	-9
	IM	-1.3420	7758	1564	-10	-3.6702	8646	0657	-10	-1.0031	6290	4111	-9	-2.7400	4864	3971	-9
20	RE	1.5258	0607	5454	-10	4.2013	4871	7991	-10	1.1566	8652	0992	-9	3.1838	6737	8116	-9
	IM	-6.0177	4321	7879	-12	-1.1432	0141	7488	-11	-1.6614	3074	9073	-11	-2.7566	3505	9871	-12

### COMPLEX FRESNEL INTEGRAL TABLE

		X															
Y		-16				-15				-14				-13			
0	RE	0.0000	0000	0000	0	0.0000	0000	0000	0	0.0000	0000	0000	0	0.0000	0000	0000	0
	IM	-1.0901	6476	0859	-8	-3.0550	2872	6056	-8	-8.5782	9779	4927	-8	-2.4141	8528	8655	-7
1	RE	9.3350	6149	5409	-9	2.6186	3306	6787	-8	7.3611	7408	5737	-8	2.0742	7067	1929	-7
	IM	-5.6120	1728	5756	-9	-1.5676	4637	6100	-8	-4.3857	2561	3424	-8	-1.2290	6375	8542	-7
2	RE	9.5955	4521	4274	-9	2.6825	4038	8042	-8	7.5114	2858	3128	-8	2.1070	7080	4829	-7
	IM	5.0937	9399	2949	-9	1.4366	7985	2212	-8	4.0631	8474	2604	-8	1.1527	6632	8538	-7
3	RE	5.8248	1862	3756	-10	1.4671	2580	4106	-9	3.5960	3308	2283	-9	8.4451	3232	0600	-9
	IM	1.0801	6219	8198	-8	3.0248	1957	4427	-8	8.4856	3197	2281	-8	2.3852	5504	3917	-7
4	RE	-8.8976	9313	6208	-9	-2.5009	6325	5295	-8	-7.0448	5370	2804	-8	-1.9892	3427	7844	-7
	IM	6.0397	3757	4807	-9	1.6720	8485	8263	-8	4.6297	1036	7744	-8	1.2818	3495	7461	-7
5	RE	-9.6979	2978	5035	-9	-2.6998	8922	1296	-8	-7.5231	0247	7931	-8	-2.0981	3657	7549	-7
	IM	-4.4619	5322	6460	-9	-1.2701	6796	3594	-8	-3.6274	4016	3274	-8	-1.0397	0743	5714	-7
6	RE	-1.1921	1839	4023	-9	-3.0394	5376	5771	-9	-7.5937	2810	7503	-9	-1.8414	6393	6518	-8
	IM	-1.0515	3156	9391	-8	-2.9392	3126	0833	-8	-8.2264	2791	0822	-8	-2.3056	1554	1733	-7
7	RE	8.2908	6273	7007	-9	2.3319	6353	7645	-8	6.5710	8398	5364	-8	1.8552	5507	7667	-7
	IM	-6.4076	8576	0321	-9	-1.7616	2089	5329	-8	-4.8388	0927	9783	-8	-1.3274	6105	4719	-7
8	RE	9.6752	5157	5922	-9	2.6827	8644	0488	-8	7.4407	1326	3991	-8	2.0639	3141	2535	-7
	IM	3.7157	9693	0173	-9	1.0674	6927	4209	-8	3.0756	9363	2328	-8	8.8899	0513	6296	-8
9	RE	1.8393	1820	6740	-9	4.7574	1891	1560	-9	1.2140	7566	8308	-8	3.0421	4055	8328	-8
	IM	1.0075	5984	4297	-8	2.8099	2664	9408	-8	7.8426	5771	3512	-8	2.1905	8475	6783	-7
10	RE	-7.5436	6478	0713	-9	-2.1216	7274	7662	-8	-5.9753	8616	8639	-8	-1.6851	9827	9124	-7
	IM	6.7365	6146	9835	-9	1.8436	7923	8353	-8	5.0391	0916	0736	-8	1.3749	3035	3407	-7
11	RE	-9.5573	5041	5508	-9	-2.6414	9645	9644	-8	-7.2994	2746	3985	-8	-2.0164	3622	4036	-7
	IM	-2.8764	8886	2158	-9	-8.3574	5181	3609	-9	-2.4331	6152	5864	-8	-7.0981	8686	9951	-8
12	RE	-2.5145	6163	3049	-9	-6.5886	5326	3064	-9	-1.7118	0542	5654	-8	-4.3999	4854	8018	-8
	IM	-9.5182	5424	1598	-9	-2.6488	9201	3914	-8	-7.3746	8511	1677	-8	-7.0537	9399	6017	-7
13	RE	6.6928	7043	9688	-9	1.8822	1348	3701	-8	5.2981	9486	1551	-8	1.4926	7103	4618	-7
	IM	-7.0306	5478	5465	-9	-1.9194	8887	2551	-8	-5.2333	5551	8814	-8	-1.4244	5919	9848	-7
14	RE	9.3620	7748	5500	-9	2.5815	2554	3062	-8	7.1157	5067	5607	-8	1.9603	8228	6671	-7
	IM	1.9759	5572	1971	-9	5.8547	0526	7377	-9	1.7348	0549	0201	-8	5.1402	4548	5208	-8
15	RE	3.1961	8081	9518	-9	8.4609	8636	8888	-9	2.2279	4914	3557	-8	5.8293	7014	5048	-8
	IM	8.8717	1816	6226	-9	2.4649	2814	5665	-8	6.8495	4159	2813	-8	1.9034	2769	6716	-7
16	RE	-5.7721	9652	4145	-9	-1.6240	4029	6903	-8	-4.5719	4745	0620	-8	-1.2877	0045	2817	-7
	IM	7.2822	0340	8499	-9	1.9861	4758	5719	-8	5.4104	1961	5663	-8	1.4717	3233	4694	-7
17	RE	-9.0964	4161	0442	-9	-2.5045	1372	0322	-8	-6.8926	7454	2790	-8	-1.8958	8606	7390	-7
	IM	-1.0466	4498	2309	-9	-3.2677	5445	2351	-9	-1.0124	2672	0838	-8	-3.1161	0934	8457	-8
18	RE	-3.8593	0385	7119	-9	-1.0295	2012	5127	-8	-2.7370	4819	5864	-8	-7.2481	5257	0219	-8
	IM	-8.1555	0024	1274	-9	-2.2635	2793	2471	-8	-6.2822	6565	6592	-8	-1.7434	1903	8864	-7
19	RE	4.8086	9865	0861	-9	1.3550	3245	1942	-8	3.8193	7972	0074	-8	1.0767	4565	7746	-7
	IM	-7.4784	5733	6494	-9	-2.0393	0081	5634	-8	-5.5553	1850	1790	-8	-1.5115	8298	6707	-7
20	RE	8.7614	8849	4892	-9	2.4101	9378	4500	-8	6.6273	6211	3636	-8	1.8213	9243	9980	-7
	IM	1.1668	5715	0785	-10	6.8022	2811	0293	-10	2.9082	8859	3350	-9	1.0988	9711	1288	-8

COMPLEX FRESNEL INTEGRAL TABLE

Y		X											
		-12			-11			-10			-9		
0	RE	0.0000	0000	0000	0	0.0000	0000	0000	0	0.0000	0000	0000	0
	IM	-6.8119	6273	5151	-7	-1.9279	2993	1304	-6	-5.4759	8795	3368	-6
1	RE	5.8612	5650	1551	-7	1.6615	9115	3623	-6	4.7284	7931	6392	-6
	IM	-3.4509	1823	6456	-7	-9.7101	1745	0070	-7	-2.7388	1605	5176	-6
2	RE	5.9225	6730	8033	-7	1.6684	9468	5898	-6	4.7124	9918	1965	-6
	IM	3.2825	1495	7768	-7	9.3873	1930	1083	-7	2.6983	6312	8390	-6
3	RE	1.8411	5844	8645	-8	3.4375	5567	2574	-8	3.8795	2371	1166	-8
	IM	6.7198	3089	5045	-7	1.8979	2773	4022	-6	5.3758	1412	9414	-6
4	RE	-5.6322	6000	8732	-7	-1.5996	2378	3013	-6	-4.5590	2353	0856	-6
	IM	3.5479	9727	5555	-7	9.8138	7092	0522	-7	2.7111	1247	2080	-6
5	RE	-5.8565	9684	8275	-7	-1.6360	6343	6416	-6	-4.5733	3609	2038	-6
	IM	-2.9921	7940	2599	-7	-8.6506	9447	3551	-7	-2.5139	1750	2147	-6
6	RE	-4.2609	7241	6158	-8	-9.0830	4649	0091	-8	-1.6262	9935	5557	-7
	IM	-6.4712	4064	6269	-7	-1.8189	8941	5251	-6	-5.1205	0915	9968	-6
7	RE	5.2490	2103	2694	-7	1.4883	7338	8828	-6	4.2300	1499	6337	-6
	IM	-3.6353	7791	1528	-7	-9.9319	0103	9912	-7	-2.7044	1352	7404	-6
8	RE	5.7247	0254	4799	-7	1.5873	8023	7161	-6	4.3987	5713	5146	-6
	IM	2.5780	7264	7540	-7	7.5024	6178	4682	-7	2.1911	3359	1608	-6
9	RE	7.4283	9088	1978	-8	1.7453	7401	9266	-7	3.8535	0312	8188	-7
	IM	6.1228	7489	0032	-7	1.7123	7596	2861	-6	4.7908	5962	6279	-6
10	RE	-4.7591	2661	2544	-7	-1.3457	7099	1383	-6	-3.8100	8788	4464	-6
	IM	3.7433	5050	9730	-7	1.0163	2577	9924	-6	2.7495	9464	7269	-6
11	RE	-5.5673	4728	7611	-7	-1.5359	1588	5958	-6	-4.2325	2781	0553	-6
	IM	-2.0748	1367	3898	-7	-6.0760	0150	7791	-7	-1.7823	1480	5742	-6
12	RE	-1.1153	4433	9413	-7	-2.7757	2226	7606	-7	-6.7358	4163	8325	-7
	IM	-5.7207	4330	0965	-7	-1.5935	3296	6307	-6	-4.4380	5444	4011	-6
13	RE	4.2086	0975	8667	-7	1.1873	9812	7298	-6	3.3516	7611	9058	-6
	IM	-3.8693	6933	1102	-7	-1.0484	9619	2834	-6	-2.8327	7915	0138	-6
14	RE	5.3970	8767	1919	-7	1.4845	1765	8637	-6	4.0786	0074	5085	-6
	IM	1.5228	4283	2515	-7	4.5101	8045	0330	-7	1.3350	7476	5165	-6
15	RE	1.5135	1806	5728	-7	3.8927	6085	4350	-7	9.8959	6063	0112	-7
	IM	5.2890	2551	3309	-7	1.4693	1396	1865	-6	4.0801	4665	6361	-6
16	RE	-3.6281	6863	9164	-7	-1.0224	8482	4322	-6	-2.8816	9236	2326	-6
	IM	3.9966	8775	5172	-7	1.0832	4493	3831	-6	2.9293	8018	9184	-6
17	RE	-5.2111	9117	0576	-7	-1.4311	8096	0484	-6	-3.9265	3580	4801	-6
	IM	-9.5354	1444	7375	-8	-2.9026	1777	1389	-7	-8.7929	1097	8007	-7
18	RE	-1.9107	6188	3660	-7	-5.0108	4139	1854	-7	-1.3060	8906	1621	-6
	IM	-4.8372	1653	0304	-7	-1.3416	6383	6290	-6	-3.7195	0661	5637	-6
19	RE	3.0357	0293	0109	-7	8.5579	9713	6477	-7	2.4120	4061	4110	-6
	IM	-4.1075	3622	4196	-7	-1.1145	0602	1843	-6	-3.0189	4326	8783	-6
20	RE	5.0025	9949	9458	-7	1.3729	9387	4531	-6	3.7650	4107	0601	-6
	IM	3.8822	3150	5751	-8	1.3143	3020	2060	-7	4.3195	6648	9672	-7

COMPLEX FRESNEL INTEGRAL TABLE

		X											
Y		-8			-7			-6			-5		
0	RE	0.0000	0000	0000	0	0.0000	0000	0000	0	0.0000	0000	0000	0
	IM	-4.4789	8997	3760	-5	-1.2926	6638	2545	-4	-3.7618	4700	3125	-4
1	RE	3.8863	7061	4029	-5	1.1251	5927	9433	-4	3.2868	4099	8996	-4
	IM	-2.1976	1373	6296	-5	-6.2563	0609	1562	-5	-1.7878	2703	5796	-4
2	RE	3.7935	5712	5524	-5	1.0820	7157	4347	-4	3.0989	7351	4344	-4
	IM	2.2744	1992	5272	-5	6.6911	6133	0388	-5	1.9923	0096	8494	-4
3	RE	-9.3143	6930	0571	-7	-5.0674	4772	2844	-6	-2.3253	3160	9029	-5
	IM	4.3570	6624	6230	-5	1.2478	6269	8620	-4	3.5901	8032	3707	-4
4	RE	-3.7496	0872	0138	-5	-1.0832	1992	2008	-4	-3.1467	2858	0951	-4
	IM	2.0525	4575	1075	-5	5.6038	2909	5908	-5	1.5149	0242	4065	-4
5	RE	-3.5757	5652	2992	-5	-9.9887	5679	5465	-5	-2.7840	1632	7872	-4
	IM	-2.1616	3153	3695	-5	-6.4045	5340	6296	-5	-1.9120	8960	2178	-4
6	RE	5.4574	1929	4174	-7	4.8011	4245	3086	-6	2.4400	8201	2981	-5
	IM	-4.0739	1567	4346	-5	-1.1507	1106	6956	-4	-3.2505	1976	3945	-4
7	RE	3.4403	0869	3842	-5	9.8421	4667	4838	-5	2.8199	7734	9766	-4
	IM	-1.9742	8860	1463	-5	-5.2705	5318	2681	-5	-1.3896	8907	6963	-4
8	RE	3.3642	1137	1903	-5	9.2707	5227	1956	-5	2.5448	9982	9111	-4
	IM	1.8890	9270	1476	-5	5.5743	6299	2990	-5	1.6491	9770	9059	-4
9	RE	1.1240	7468	2304	-6	3.5707	4876	8038	-8	-9.5692	1531	2723	-6
	IM	3.7500	9846	5325	-5	1.0482	4674	5136	-4	2.9257	5534	7661	-4
10	RE	-3.0623	6885	4038	-5	-8.6878	9262	5335	-5	-2.4641	3670	8936	-4
	IM	1.9831	4433	3822	-5	5.2725	1193	8045	-5	1.3890	4175	2948	-4
11	RE	-3.1978	4421	8595	-5	-8.7579	8981	6580	-5	-2.3905	7949	2987	-4
	IM	-1.5394	7995	0184	-5	-4.5289	7348	4812	-5	-1.3321	1068	8089	-4
12	RE	-3.4899	3099	6300	-6	-7.0188	1175	8894	-6	-1.1491	6458	1093	-5
	IM	-3.4366	6725	3979	-5	-9.5484	6803	0607	-5	-2.6485	9324	6952	-4
13	RE	2.6717	4998	0227	-5	7.5402	1235	7339	-5	2.1262	5487	9812	-4
	IM	-2.0445	6623	0492	-5	-5.4539	0935	8166	-5	-1.4463	0226	9035	-4
14	RE	3.0644	2880	2578	-5	8.3746	0091	1911	-5	2.2828	9814	0943	-4
	IM	1.1666	2627	5443	-5	3.4412	4115	5513	-5	1.0129	9899	3354	-4
15	RE	6.0941	1381	7145	-6	1.4611	6257	5036	-5	3.3847	8218	7797	-5
	IM	3.1396	9001	3624	-5	8.6958	7691	3776	-5	2.4050	1639	6100	-4
16	RE	-2.2872	8450	3693	-5	-6.4384	2440	6693	-5	-1.8105	7831	6233	-4
	IM	2.1249	2276	5013	-5	5.6954	8954	7103	-5	1.5208	4660	7136	-4
17	RE	-2.9442	4899	1467	-5	-8.0434	2307	0256	-5	-2.1933	1676	0632	-4
	IM	-7.9593	2158	1638	-6	-2.3787	5658	1253	-5	-7.0775	1164	3460	-5
18	RE	-8.6761	1345	0446	-6	-2.2051	2514	1748	-5	-5.5393	7438	7950	-5
	IM	-2.8527	9182	8010	-5	-7.8896	6288	9481	-5	-2.1793	8161	8231	-4
19	RE	1.9133	7952	7826	-5	5.3831	1970	4940	-5	1.5129	5758	0966	-4
	IM	-2.2022	8077	4047	-5	-5.9284	3746	6344	-5	-1.5919	4398	5333	-4
20	RE	2.8224	8883	6434	-5	7.7139	4794	6711	-5	2.1053	1993	5228	-4
	IM	4.3874	2498	7607	-6	1.3668	0482	3905	-5	4.2075	2787	2666	-5

COMPLEX FRESNEL INTEGRAL TABLE

Y		X									
		-4		-3		-2		-1			
0	RE	0.0000	0000 0000	0	0.0000	0000 0000	0	0.0000	0000 0000	0	0
	IM	-3.3076	5812 5692	-3	-1.0115	7836 5252	-2	-3.2173	5451 4689	-2	-1
1	RE	2.9198	0620 9576	-3	8.9866	4359 1980	-3	2.8703	6042 3974	-2	-2
	IM	-1.4753	6801 0522	-3	-4.2375	4185 8084	-3	-1.1924	1037 5461	-2	-2
2	RE	2.5715	2067 1404	-3	7.4195	8689 8416	-3	2.1156	0189 4931	-2	-2
	IM	1.8606	6231 4800	-3	5.9072	8068 8627	-3	1.9432	3412 2180	-2	-2
3	RE	-4.1338	7304 6730	-4	-1.6984	0098 0712	-3	-6.9429	3661 8955	-3	-2
	IM	3.0124	9747 6542	-3	8.7591	2566 0126	-3	2.5328	4994 1152	-2	-2
4	RE	-2.7009	3589 8869	-3	-7.9613	1951 2395	-3	-2.3430	5444 6209	-2	-2
	IM	1.0403	2096 4700	-3	2.5500	6939 6591	-3	5.6480	7916 8862	-3	-3
5	RE	-2.1262	9079 3662	-3	-5.7737	4953 6521	-3	-1.5341	6415 1800	-2	-2
	IM	-1.7432	4271 7985	-3	-5.3080	7427 9728	-3	-1.6167	5562 7844	-2	-2
6	RE	4.1690	6618 6193	-4	1.5700	7008 3796	-3	5.6762	7726 8536	-3	-2
	IM	-2.5800	0209 2319	-3	-7.2151	0783 9604	-3	-1.9982	8374 3056	-2	-2
7	RE	2.3164	6542 5134	-3	6.6188	7545 5656	-3	1.8811	3877 4939	-2	-2
	IM	-9.0999	0265 8266	-4	-2.2188	2378 0986	-3	-5.1321	5034 2405	-3	-2
8	RE	1.8831	8040 3048	-3	5.0548	7465 7917	-3	1.3403	9034 7705	-2	-2
	IM	1.4485	8467 2073	-3	4.2859	1959 1959	-3	1.2626	5224 8196	-2	-2
9	RE	-2.4958	7770 3940	-4	-9.7129	3158 6704	-4	-3.5144	6854 2976	-3	-2
	IM	2.2602	7757 9680	-3	6.2408	9383 3339	-3	1.7124	1752 3858	-2	-2
10	RE	-1.9746	5643 3805	-3	-5.5681	1201 8564	-3	-1.5636	3063 1738	-2	-2
	IM	9.2872	7137 6394	-4	2.3419	1531 0674	-3	5.7755	7274 3448	-3	-2
11	RE	-1.7576	2397 1667	-3	-4.7258	0440 4034	-3	-1.2620	5857 3457	-2	-2
	IM	-1.1476	7731 9301	-3	-3.3544	6488 5.91	-3	-9.7613	4278 0117	-3	-2
12	RE	4.7043	6381 9671	-5	3.3241	2387 8320	-4	1.4969	0072 2934	-3	-3
	IM	-2.0233	5012 6089	-3	-5.5657	0059 4621	-3	-1.5248	7461 3172	-2	-2
13	RE	1.6831	9107 3520	-3	4.7198	9317 3518	-3	1.3195	4072 9981	-2	-2
	IM	-9.9529	8523 9219	-4	-2.5771	6522 3583	-3	-6.6052	0114 7867	-3	-2
14	RE	1.6808	9308 1615	-3	4.5365	0344 7127	-3	1.2193	5961 9370	-2	-2
	IM	8.7050	1636 2566	-4	2.5381	6287 0147	-3	7.3683	4038 2282	-3	-2
15	RE	1.5123	8296 6524	-4	2.6173	9177 3240	-4	2.8307	7104 4127	-4	-4
	IM	1.8293	8122 4365	-3	5.0281	0571 7868	-3	1.3782	6949 3234	-2	-2
16	RE	-1.4257	9507 1805	-3	-3.9899	1945 9921	-3	-1.1139	6409 3197	-2	-2
	IM	1.0704	3432 0063	-3	2.8191	8246 2509	-3	7.3850	3203 5373	-3	-2
17	RE	-1.6203	9609 9093	-3	-4.3883	7232 4090	-3	-1.1853	1747 1443	-2	-2
	IM	-6.1791	0730 1987	-4	-1.8126	4210 5044	-3	-5.2907	2281 5294	-3	-2
18	RE	-3.3396	5995 2960	-4	-7.9531	4219 8398	-4	-1.8405	3464 9210	-3	-3
	IM	-1.6557	6737 1333	-3	-4.5522	3190 1533	-3	-1.2491	3765 7139	-2	-2
19	RE	1.1904	3160 6740	-3	3.3311	7705 6115	-3	9.3040	8196 4855	-3	-2
	IM	-1.1384	7536 9903	-3	-3.0309	6073 4752	-3	-8.0437	7707 6305	-3	-2
20	RE	1.5608	7333 0075	-3	4.2391	1704 4700	-3	1.1491	6763 6052	-2	-2
	IM	3.8696	0752 9519	-4	1.1583	6926 6485	-3	3.4408	8651 9616	-3	-2

COMPLEX FRESNEL INTEGRAL TABLE

Y		X											
		0			1			2			3		
0	RE	0.0000	0000	0000	0	1.1670	2724	5891	0	2.6680	0051	4199	0
	IM	-7.0710	6781	1865	-1	-7.0710	6781	1865	-1	-7.0710	6781	1865	-1
1	RE	3.3527	3009	1524	-1	8.9881	0821	2496	-1	1.9353	5154	1318	0
	IM	-2.1733	4841	5936	-2	2.5775	1280	0297	-1	1.1485	4842	2256	0
2	RE	1.3467	0939	5986	-1	2.3678	1740	5965	-1	2.6714	7889	8607	-1
	IM	2.2355	2715	1694	-1	6.8455	1536	3876	-1	1.8495	8217	5631	0
3	RE	-1.0653	6025	8436	-1	-3.6973	1351	2385	-1	-1.1511	6478	2435	0
	IM	1.9283	1802	6650	-1	4.8194	1044	6003	-1	1.1222	5178	3290	0
4	RE	-1.9369	4762	0583	-1	-5.3049	5229	4960	-1	-1.3976	4566	6881	0
	IM	7.2914	8149	8866	-3	-4.1810	3482	5948	-2	-2.7271	3016	5011	-1
5	RE	-9.7216	4852	3014	-2	-2.2723	3888	2487	-1	-5.0231	0442	8147	-1
	IM	-1.4538	2482	4174	-1	-4.2148	4822	8071	-1	-1.1864	8611	0730	0
6	RE	6.6058	9697	8309	-2	2.1197	3162	3952	-1	6.5282	5838	0112	-1
	IM	-1.4628	1851	4916	-1	-3.8405	3167	7035	-1	-9.8775	6852	5513	-1
7	RE	1.4772	9871	0093	-1	4.0612	7401	2159	-1	1.1007	1158	9759	0
	IM	-2.0286	5152	2258	-2	-2.7317	0658	4625	-2	2.0852	4613	3508	-3
8	RE	9.0008	7743	8167	-2	2.2723	6831	6978	-1	5.6365	7915	6131	-1
	IM	1.0699	2939	6973	-1	3.0623	6521	7334	-1	8.6502	1253	8278	-1
9	RE	-3.9887	6219	6961	-2	-1.2724	1922	3960	-1	-3.9388	3900	1988	-1
	IM	1.2587	7436	6365	-1	3.3667	1475	1916	-1	8.9220	3092	3198	-1
10	RE	-1.2124	9230	5957	-1	-3.3423	2243	1084	-1	-9.1461	0311	9584	-1
	IM	3.2102	7976	2719	-2	7.1110	5589	0886	-2	1.4805	9136	7843	-1
11	RE	-8.7960	1710	0870	-2	-2.2940	0298	4341	-1	-5.9350	3244	8718	-1
	IM	-8.1191	2659	7952	-2	-2.3163	3114	7715	-1	-6.5556	6326	2966	-1
12	RE	2.0326	7275	3168	-2	6.7949	2013	2166	-2	2.1825	6744	5264	-1
	IM	-1.1287	6672	7216	-1	-3.0478	4748	5964	-1	-8.1874	8098	1953	-1
13	RE	1.0199	4984	4423	-1	2.8177	3164	3449	-1	7.7491	3049	4352	-1
	IM	-4.1876	1840	9726	-2	-1.0333	7113	6098	-1	-2.5117	4257	7136	-1
14	RE	8.6939	8658	3485	-2	2.3057	1788	4693	-1	6.0877	6648	4487	-1
	IM	6.1157	9345	5473	-2	1.7471	5562	6563	-1	4.9616	6947	2101	-1
15	RE	-4.5730	0869	9997	-3	-2.1673	2327	5922	-2	-8.3837	2181	4581	-2
	IM	1.0262	9003	5344	-1	2.7870	4861	3140	-1	7.5438	9993	6705	-1
16	RE	-8.6136	8090	5979	-2	-2.3845	5367	3285	-1	-6.5803	9328	2435	-1
	IM	4.9806	6760	8657	-2	1.2817	6560	8776	-1	3.2781	7656	4080	-1
17	RE	-8.5753	5242	0567	-2	-2.2966	7481	8322	-1	-6.1337	1184	6686	-1
	IM	-4.4374	7484	8550	-2	-1.2749	1054	7921	-1	-3.6433	6652	5456	-1
18	RE	-8.5928	9015	0141	-3	-1.6343	1363	9393	-2	-2.5258	7965	1813	-2
	IM	-9.3460	7798	5758	-2	-2.5479	0528	5738	-1	-6.9301	6799	4130	-1
19	RE	7.2121	6384	1379	-2	2.0010	6523	8646	-1	5.5386	3714	0229	-1
	IM	-5.6095	9189	3631	-2	-1.4739	7560	8276	-1	-3.8602	0179	1635	-1
20	RE	8.3964	1171	2365	-2	2.2629	3910	6782	-1	6.0870	6008	0265	-1
	IM	2.9723	0573	5092	-2	8.6503	2734	8517	-2	2.5021	7690	0936	-1

COMPLEX FRESNEL INTEGRAL TABLE

		X											
Y		4			5			6			7		
0	RE	1.3127	2976	7873	1	3.0637	2608	1069	1	7.3740	0002	1904	1
	IM	-7.0710	6781	1865	-1	-7.0710	6781	1865	-1	-7.0710	6781	1865	-1
1	RE	8.6870	4804	2150	0	1.9551	7904	4409	1	4.5760	4347	9457	1
	IM	8.7984	9514	3582	0	2.2266	5088	7052	1	5.6052	7163	8605	1
2	RE	-1.1756	7252	1223	0	-4.8728	0380	2912	0	-1.5484	5617	0345	1
	IM	1.1343	3936	3146	1	2.7682	6650	8202	1	6.8121	1642	4829	1
3	RE	-8.9057	8607	7804	0	-2.3432	4676	5160	1	-6.0845	5467	7222	1
	IM	5.5104	9411	5124	0	1.2214	4217	8230	1	2.7540	6949	4949	1
4	RE	-8.9527	4903	9712	0	-2.2281	0306	0328	1	-5.5447	4955	0395	1
	IM	-3.6017	2141	5068	0	-1.0861	5002	8347	1	-3.1055	5010	2125	1
5	RE	-2.0870	9451	8257	0	-3.9149	1209	4069	0	-6.8164	3171	7065	0
	IM	-8.7014	2901	1088	0	-2.2955	4225	7477	1	-6.0008	0432	5723	1
6	RE	5.5640	3885	7468	0	1.5594	0820	7981	1	4.2903	1329	3709	1
	IM	-6.2318	6777	0426	0	-1.5445	4236	3191	1	-3.8185	8033	5108	1
7	RE	7.7835	0736	9491	0	2.0406	5870	5356	1	5.3194	1068	4465	1
	IM	1.0198	6965	7476	0	3.8534	8248	8725	0	1.2844	3531	1212	1
8	RE	3.3102	5647	9490	0	7.8770	8298	7028	0	1.8587	1505	6360	1
	IM	6.6388	8272	1386	0	1.8085	6432	8480	1	4.8844	3083	1643	1
9	RE	-3.4984	3649	9643	0	-1.0106	6316	2785	1	-2.8722	8136	0393	1
	IM	6.1154	1918	7906	0	1.5861	3659	7232	1	4.0971	6287	8502	1
10	RE	-6.7053	6335	3091	0	-1.7988	1444	3916	1	-4.8017	8598	3360	1
	IM	4.4220	9202	2295	-1	3.9184	8721	9897	-1	-9.3598	8409	0070	-1
11	RE	-3.8879	0133	0550	0	-9.8629	7267	3913	0	-2.4911	2403	8999	1
	IM	-5.1256	7808	6787	0	-1.4170	8565	6095	1	-3.8918	3460	4614	1
12	RE	2.0644	9463	7037	0	6.1428	6166	0082	0	1.7966	8383	6880	1
	IM	-5.8238	6706	7133	0	-1.5435	7602	0512	1	-4.0775	0077	3739	1
13	RE	5.7821	2416	0843	0	1.5694	4651	6641	1	4.2439	5537	5035	1
	IM	-1.4091	4277	2975	0	-3.2378	2511	0689	0	-7.2549	9464	8171	0
14	RE	4.1918	6257	5412	0	1.0940	8320	6238	1	2.8472	5587	8177	1
	IM	3.9304	8324	0640	0	1.0967	6340	2460	1	3.0441	7794	3407	1
15	RE	-9.7186	7408	2356	-1	-3.0856	2344	6917	0	-9.5116	6791	7940	0
	IM	5.4749	3493	2097	0	1.4684	9538	7285	1	3.9287	5244	1642	1
16	RE	-4.9633	7143	2647	0	-1.3567	9702	8562	1	-3.6981	8254	0946	1
	IM	2.1046	9381	5227	0	5.2843	2106	3648	0	1.3188	9552	0070	1
17	RE	-4.3401	6143	6745	0	-1.1503	3488	1219	1	-3.0424	9955	6426	1
	IM	-2.9290	4665	3648	0	-8.2426	3471	5796	0	-2.3086	5412	3839	1
18	RE	9.2441	7917	4288	-2	6.1688	1392	8571	-1	2.6331	5385	6246	0
	IM	-5.0924	8836	7635	0	-1.3760	1680	1831	1	-3.7106	9843	5032	1
19	RE	4.2118	1404	1248	0	1.1572	0540	7936	1	3.1719	6667	4207	1
	IM	-2.6223	2666	0269	0	-6.8040	3779	6829	0	-1.7605	4827	2623	1
20	RE	4.3796	7031	3979	0	1.1717	0825	9836	1	3.1297	3829	2082	1
	IM	2.0580	9426	1209	0	5.8558	7635	9647	0	1.6580	5452	4918	1



COMPLEX FRESNEL INTEGRAL TABLE

		X											
Y		8			9			10			11		
0	RE	4.5475	9370	6091	2	1.1525	8025	0939	3	2.9476	0027	5872	3
	IM	-7.0710	6781	1865	-1	-7.0710	6781	1865	-1	-7.0710	6781	1865	-1
1	RE	2.7201	1164	7563	2	6.8111	3877	6713	2	1.7254	0951	7906	3
	IM	3.6037	8502	7279	2	9.2278	4464	5860	2	2.3767	6889	9043	3
2	RE	-1.2414	4841	2440	2	-3.3719	4881	8795	2	-9.0626	5840	3491	2
	IM	4.2604	1791	4075	2	1.0808	6785	4792	3	2.7631	3633	6770	3
3	RE	-4.0637	5467	9903	2	-1.0519	7927	3095	3	-2.7304	0955	4154	3
	IM	1.4878	5210	8625	2	3.5556	5516	4020	2	8.6278	5871	5954	2
4	RE	-3.4825	4401	5860	2	-8.8099	7967	0417	2	-2.2425	4122	3101	3
	IM	-2.3432	7287	1716	2	-6.3103	7928	2139	2	-1.6887	8495	2908	3
5	RE	-1.1489	5603	1219	1	3.7979	1436	1624	0	8.3068	2880	5502	1
	IM	-4.0561	6475	0576	2	-1.0539	2163	6592	3	-2.7418	9572	3162	3
6	RE	3.1352	3285	0514	2	8.3881	2763	0989	2	2.2358	7606	5501	3
	IM	-2.3462	8910	2275	2	-5.8521	1904	0585	2	-1.4671	8143	1516	3
7	RE	3.5852	5183	2214	2	9.3003	6739	7405	2	2.4145	9793	5738	3
	IM	1.1880	0476	0984	2	3.4341	0254	7976	2	9.7308	1870	3736	2
8	RE	1.0188	4806	8282	2	2.3751	8714	7017	2	5.5295	7710	7243	2
	IM	3.4983	5275	5856	2	9.3079	7194	5311	2	2.4710	6382	1719	3
9	RE	-2.2345	4997	5991	2	-6.1470	3738	3089	2	-1.6798	7714	6406	3
	IM	2.7168	0623	2310	2	6.9910	3293	6295	2	1.8002	3396	2478	3
10	RE	-3.3849	4210	2509	2	-8.9554	4241	5070	2	-2.3661	5694	5965	3
	IM	-3.1516	6568	5194	1	-1.1244	5109	9569	2	-3.6745	3261	5090	2
11	RE	-1.5757	2572	9158	2	-3.9544	9198	7151	2	-9.9216	8043	4761	2
	IM	-2.8873	9481	3067	2	-7.8136	7666	4955	2	-2.1075	7915	5145	3
12	RE	1.4756	6001	9162	2	4.1618	8050	7399	2	1.1640	7363	6137	3
	IM	-2.8245	4561	3742	2	-7.4164	1122	6945	2	-1.9457	0041	2967	3
13	RE	3.0739	8859	5217	2	8.2418	1574	0885	2	2.2055	2508	7531	3
	IM	-3.2757	4125	0546	1	-6.3875	3527	7224	1	-1.1125	8831	9065	2
14	RE	1.9156	8621	8120	2	4.9580	5221	9306	2	1.2821	5804	8503	3
	IM	2.3121	6489	1325	2	6.3333	4642	3156	2	1.7288	6161	2638	3
15	RE	-8.4832	8773	2304	1	-2.4744	6692	3834	2	-7.1317	8187	5203	2
	IM	2.7941	0488	4210	2	7.4325	9176	4733	2	1.9746	7684	3295	3
16	RE	-2.7259	4812	1097	2	-7.3754	8800	5381	2	-1.9916	9383	6559	3
	IM	8.0750	6102	3351	1	1.9812	4629	0537	2	4.8337	4984	2827	2
17	RE	-2.1172	9459	9954	2	-5.5740	7216	1471	2	-1.4660	0740	2566	3
	IM	-1.7877	9747	6030	2	-4.9461	8401	0904	2	-1.3637	5224	2201	3
18	RE	3.2546	8695	8397	1	1.0464	8748	4869	2	3.2530	8610	1294	2
	IM	-2.6842	0780	7611	2	-7.2028	1638	8408	2	-1.9303	4797	6249	3
19	RE	2.3674	9770	1278	2	6.4484	1793	1453	2	1.7531	9307	8656	3
	IM	-1.1700	3147	5109	2	-3.0065	6878	9620	2	-7.7116	9185	8962	2
20	RE	2.2236	6817	5675	2	5.9167	1796	7301	2	1.5727	9739	9696	3
	IM	1.3115	8801	2546	2	3.6665	9032	0339	2	1.0212	9545	2680	3

COMPLEX FRESNEL INTEGRAL TABLE

Y		12		13		14		15	
0	RE	1.9653 7019 1300	4	5.1112 3401 0358	4	1.3341 6488 6584	5	3.4933 0155 5588	5
	IM	-7.0710 6781 1865	-1	-7.0710 6781 1865	-1	-7.0710 6781 1865	-1	-7.0710 6781 1865	-1
1	RE	1.1365 3072 6472	4	2.9351 1075 9081	4	7.6272 7170 6175	4	1.9894 1031 6858	5
	IM	1.5974 4654 9641	4	4.1730 5039 8904	4	1.0921 3993 0630	5	2.8658 8666 7930	5
2	RE	-6.4556 5080 5945	3	-1.7186 2793 5339	4	-4.5722 1447 9804	4	-1.2163 2686 1553	5
	IM	1.8282 6353 5890	4	4.7746 7579 2829	4	1.2447 5286 1139	5	3.2553 0474 5280	5
3	RE	-1.8550 0059 7010	4	-4.8543 0355 4034	4	-1.2733 6445 3869	5	-3.3473 7475 9890	5
	IM	5.2680 9185 7642	3	1.3205 5622 2424	4	3.3354 2686 4127	4	8.4790 1299 5862	4
4	RE	-1.4770 0324 2923	4	-3.8170 5581 4293	4	-9.9030 2154 9657	4	-2.5780 5607 9139	5
	IM	-1.1986 3299 9687	4	-3.1873 0732 3178	4	-8.4728 0258 9159	4	-2.2524 7590 3494	5
5	RE	1.3437 1300 4323	3	4.3219 3592 0603	3	1.3165 1481 6371	4	3.8797 3538 5874	4
	IM	-1.8652 9150 5959	4	-4.8840 5076 0534	4	-1.2807 4909 0857	5	-3.3649 1984 4371	5
6	RE	1.5796 5805 2766	4	4.1959 8577 0135	4	1.1134 1854 4803	5	2.9565 0554 9675	5
	IM	-9.3679 4624 8197	3	-2.3047 4407 9625	4	-6.0979 4720 9502	4	-1.5656 8898 3451	5
7	RE	1.6349 5931 3836	4	4.2660 6648 9012	4	1.1152 6868 7837	5	2.9211 1957 1536	5
	IM	7.5202 8951 0143	3	2.0649 2316 1451	4	5.6394 0247 1921	4	1.5340 0697 8099	5
8	RE	2.9898 9032 1301	3	6.9441 1979 1931	3	1.6107 1137 0581	4	3.7287 0277 6629	4
	IM	1.7358 1265 5083	4	4.5981 4362 6365	4	1.2182 0403 9377	5	3.2285 8807 5586	5
9	RE	-1.2370 9245 2783	4	-3.3407 7758 3089	4	-9.0019 7029 0468	4	-2.4216 9367 3249	5
	IM	1.1987 6049 0042	4	3.1017 8796 0338	4	8.0419 2922 2931	4	2.0892 5870 6582	5
10	RE	-1.6489 1118 6590	4	-4.3526 0177 6302	4	-1.1493 3405 5716	5	-3.0364 6522 2919	5
	IM	-3.4194 5758 8321	3	-1.0016 7014 1557	4	-2.8844 0170 2092	4	-8.2002 4336 7469	4
11	RE	-6.2566 3438 8239	3	-1.5730 9483 5704	4	-3.9661 5124 1558	4	-1.0012 0306 1034	5
	IM	-1.5224 0659 4504	4	-4.0814 7214 2593	4	-1.0930 0748 3268	5	-2.9247 5747 7155	5
12	RE	8.9311 7475 1683	3	2.4552 9330 7453	4	6.7239 7944 1267	4	1.8355 9849 2319	5
	IM	-1.3381 7579 8819	4	-3.5104 2791 3327	4	-9.2136 9563 9880	4	-2.4199 5453 0435	5
13	RE	1.5728 6500 2321	4	4.1944 6526 7293	4	1.1179 4156 9805	5	2.9786 1229 3687	5
	IM	-3.0862 7754 6883	1	8.2590 9777 2034	2	4.4467 2568 8014	3	1.7399 0503 6733	4
14	RE	8.5674 7461 9191	3	2.2152 8040 5473	4	5.7311 1797 2377	4	1.4837 8796 6584	5
	IM	1.2774 8062 3856	4	3.4608 4794 1401	4	9.3592 5360 2063	4	2.5273 4160 1552	5
15	RE	-5.7595 9035 6124	3	-1.6186 6189 7882	4	-4.5223 8694 2268	4	-1.2572 4866 1328	5
	IM	1.3902 4433 3025	4	3.6855 6809 4156	4	9.7703 6183 8900	4	2.5896 6285 6810	5
16	RE	-1.4454 7642 3789	4	-3.8866 6056 5051	4	-1.0440 3502 9701	5	-2.8022 6002 5011	5
	IM	2.8272 0192 2920	3	6.7728 5742 4256	3	1.6109 2771 8732	4	3.8004 6088 8553	4
17	RE	-1.0121 0590 6587	4	-2.6579 7536 1962	4	-6.9798 0982 0034	4	-1.8330 5477 6496	5
	IM	-1.0275 2725 2270	4	-2.8097 3263 1173	4	-7.6666 1267 1339	4	-2.0879 7436 2853	5
18	RE	2.9369 7968 3844	3	8.6146 3636 4227	3	2.4970 6461 5830	4	7.1682 7487 2542	4
	IM	-1.3821 8501 1267	4	-3.6941 5536 9758	4	-9.8676 0779 8031	4	-2.6346 1597 2119	5
19	RE	1.2897 6314 5763	4	3.4910 5526 4676	4	9.4384 2348 6301	4	2.5491 9002 2604	5
	IM	-5.0503 6292 9256	3	-1.2900 4001 9765	4	-3.2919 8750 5598	4	-8.3938 3902 5141	4
20	RE	1.1088 6084 8843	4	2.9418 4931 8751	4	7.8019 7950 7022	4	2.0686 3550 2145	5
	IM	7.8473 5854 2932	3	2.1659 8234 3767	4	5.9635 5132 3366	4	1.6382 5299 0240	5

COMPLEX FRFSNEL INTEGRAL TABLE

		X											
Y		16			17			18			19		
0	RE	9.1708	9020	5163	5	2.4131	3607	9793	6	6.3624	2406	2043	6
	IM	-7.0710	6781	1865	-1	-7.0710	6781	1865	-1	-7.0710	6781	1865	-1
1	RE	5.2052	7903	0184	5	1.3656	3444	1218	6	3.5912	1063	2685	6
	IM	7.5378	1618	4661	5	1.9866	1882	2478	6	5.2452	2137	8123	6
2	RE	-3.2364	8756	0725	5	-8.6152	4729	7804	5	-2.2944	1146	5117	6
	IM	8.5365	2519	3631	5	2.2438	7598	1012	6	5.9104	3513	3595	6
3	RE	-8.8165	4035	1335	5	-2.3262	2812	6278	6	-6.1474	3181	4312	6
	IM	2.1674	0522	4843	5	5.5668	6969	1133	5	1.4357	9052	0266	6
4	RE	-6.7316	7620	1151	5	-1.7624	1224	6664	6	-4.6250	3340	8156	6
	IM	-5.9099	5197	3140	5	-1.5935	9015	7315	6	-4.2418	4198	7665	6
5	RE	1.1187	4431	9144	5	3.1776	9833	6983	5	8.9285	3674	2966	5
	IM	-8.8565	0434	6400	5	-2.3349	1097	9953	6	-6.1651	4517	1623	6
6	RE	7.8536	1040	5171	5	2.0872	7561	4436	6	5.5505	8459	7497	6
	IM	-4.0350	3078	2372	5	-1.0434	1603	5811	6	-2.7064	4984	6534	6
7	RE	7.6648	9637	8566	5	2.0146	9887	0741	6	5.3041	5775	1168	6
	IM	4.1603	1957	9298	5	1.1257	8397	4751	6	3.0412	5946	0080	6
8	RE	8.6065	3752	9066	4	1.9783	1919	8878	5	4.5213	8394	4819	5
	IM	8.5609	9962	4585	5	2.2714	1208	0460	6	6.0304	6142	4241	6
9	RE	-6.5070	3190	5978	5	-1.7469	2934	7110	6	-4.6871	5255	5798	6
	IM	5.4387	5612	0550	5	1.4185	9452	0559	6	3.7071	0228	0647	6
10	RE	-8.0272	8631	5243	5	-2.1236	4581	7369	6	-5.6224	8699	9541	6
	IM	-2.3085	8745	9894	5	-6.4499	6797	5741	5	-1.7912	5538	9010	6
11	RE	-2.5322	9182	9544	5	-6.4175	8344	1233	5	-1.6296	8866	4024	6
	IM	-7.8221	9779	5100	5	-2.0913	5666	4376	6	-5.5905	7809	7032	6
12	RE	4.9981	0175	1268	5	1.3580	4048	8684	6	3.6835	6699	2370	6
	IM	-6.3610	8832	5193	5	-1.6735	5793	8719	6	-4.4071	3114	7090	6
13	RE	7.9348	4571	1576	5	2.1137	5432	3234	6	5.6313	2853	1674	6
	IM	6.0080	4317	9599	4	1.9403	6224	9043	5	6.0117	1662	8420	5
14	RE	3.8449	7285	0316	5	9.9736	3637	8205	5	2.5899	3003	3047	6
	IM	6.8165	7458	9045	5	1.8367	3644	2340	6	4.9452	6898	6388	6
15	RE	-3.4805	3464	1381	5	-9.6010	3374	3967	5	-2.6403	9652	7906	6
	IM	6.8645	3559	9459	5	1.8199	7154	6502	6	4.8266	2368	4301	6
16	RE	-7.5167	3140	2476	5	-2.0153	0062	4603	6	-5.4012	5987	3627	6
	IM	8.8804	9015	5500	4	2.0511	8176	7217	5	4.6696	1005	2026	5
17	RE	-4.8151	2053	6331	5	-1.2652	8745	1580	6	-3.3263	0995	9812	6
	IM	-5.6772	1663	7923	5	-1.5414	4971	5030	6	-4.1801	4667	4652	6
18	RE	2.0413	1547	3215	5	5.7740	5854	9925	5	1.6239	7833	8996	6
	IM	-7.0321	1123	4626	5	-1.8765	6980	1189	6	-5.0072	0705	1751	6
19	RE	6.8789	8657	9471	5	1.8549	0121	3141	6	4.9985	1159	9178	6
	IM	-2.1388	2197	0567	5	-5.4469	7367	6560	5	-1.3865	9300	6436	6
20	RE	5.4841	1115	4512	5	1.4538	3272	0621	6	3.8543	0407	2967	6
	IM	4.4914	0889	0340	5	1.2291	4096	1160	6	3.3583	2233	9284	6

COMPLEX FRESNEL INTEGRAL TABLE

X

Y		20	
0	RE	4.4455 3789 9051	7
	IM	-7.0710 6781 1865	-1
1	RE	2.4981 6129 7490	7
	IM	3.6734 5206 6040	7
2	RE	-1.6299 6803 5960	7
	IM	4.1225 8822 4053	7
3	RE	-4.3111 0591 6751	7
	IM	9.6522 5215 6392	6
4	RE	-3.2045 6726 4630	7
	IM	-3.0105 4840 5258	7
5	RE	6.8940 2668 2847	6
	IM	-4.3159 7395 8725	7
6	RE	3.9323 4955 9769	7
	IM	-1.8356 7922 9789	7
7	RE	3.6927 9527 1633	7
	IM	2.2117 7656 6480	7
8	RE	2.3004 6505 6304	6
	IM	4.2593 9663 0155	7
9	RE	-3.3707 5109 8814	7
	IM	2.5449 5777 8729	7
10	RE	-3.9504 9307 3202	7
	IM	-1.3630 2731 9071	7
11	RE	-1.0572 3091 9803	7
	IM	-3.9948 7483 1240	7
12	RE	2.6993 1377 5292	7
	IM	-3.0650 5319 8798	7
13	RE	3.9993 3086 3599	7
	IM	5.3461 8830 3222	6
14	RE	1.7525 2472 9485	7
	IM	3.5787 8553 5038	7
15	RE	-1.9822 5617 0415	7
	IM	3.3985 5245 3783	7
16	RE	-3.8771 8590 8823	7
	IM	2.2725 0506 7330	6
17	RE	-2.3025 4652 3935	7
	IM	-3.0648 6825 8793	7
18	RE	1.2669 7374 1261	7
	IM	-3.5649 1043 4549	7
19	RE	3.6243 0439 9329	7
	IM	-8.9767 7391 0743	6
20	RE	2.7102 1361 9640	7
	IM	2.4967 0779 7292	7

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